1. Let A be the ratio of the volume of a sphere to the volume of a cube each of whose faces is tangent to the sphere, and let B be the ratio of the surface area of this sphere to the surface area of the cube. Then

   A. $\dfrac{A}{B} > 1$ B. $\dfrac{A}{B} = 1$ C. $\dfrac{A}{B} > \dfrac{1}{2}$

   D. $\dfrac{A}{B} = \dfrac{1}{2}$ E. $\dfrac{A}{B} < \dfrac{1}{2}$

2. The polynomial $p(x) = 2x^4 - x^3 - 7x^2 + ax + b$ is divisible by $x^2 - 2x - 3$ for certain values of a and b. What is the sum of a and b?

   A. -34 B. -30 C. -26

   D. -18 E. 30

3. Which of the following best describes the solution(s) of $3x^2 - 6x = 7$?

   A. 2 integers

   B. 2 rationals

   C. 2 irrationals

   D. 2 complex numbers

   E. none of these

4. The product of $\dfrac{x^3 + x^2 - x - 1}{x^2 - 1}$ and $\dfrac{x^2 - 2x + 1}{x^2 + 2x + 1}$ is
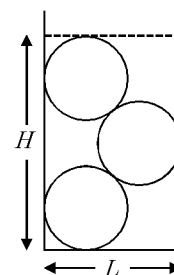
   A. $\dfrac{x - 1}{x + 1}$ B. $\dfrac{(x - 1)^2}{x + 1}$

   C. $\dfrac{x^2 + x - 1}{x - 1}$ D. $\dfrac{1}{x + 1}$

   E. none of these

5. The equation $x^3 + x^2 + 23x + 23 = 0$ has

   A. 3 real roots

   B. 3 imaginary roots

   C. 2 real roots and 1 imaginary roots

   D. 2 imaginary roots and 1 real root

   E. no real or imaginary roots

6. Each coin has radius R. The height H of the top of the highest coin above the bottom of the container is:

   A. $2R \arctan\left(\dfrac{L}{R}\right) + 2R$

   B. $4R \arctan\left(\dfrac{L}{R}\right)$

   C. $\sqrt{4LR - L^2}$

   D. $\sqrt{4LR - L^2} + R$

   E. $2\sqrt{4LR - L^2} + 2R$

7. In the given right triangle, angle 1 equals twice angle 3 and angle 5 is greater than twice angle 4. Therefore, which one of the following must be true?
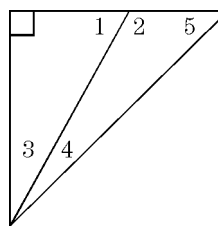
   A. angle 1 is greater than the sum of angles 4 and 5

   B. angle 3 is greater than angle 4

   C. angle 3 equals angle 4

   D. angle 4 is greater than angle 3

   E. angle 5 equals twice angle 3

8. $\overline{UV}$ is parallel to $\overline{WX}$. Which proportion is not true? Use the figure.
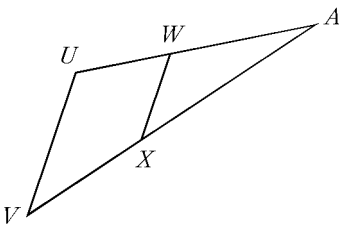
A. $\dfrac{UV}{WX} = \dfrac{UA}{WA}$

B. $\dfrac{UW}{VX} = \dfrac{WA}{XA}$

C. $\dfrac{UW}{WA} = \dfrac{UV}{WX}$

D. $\dfrac{AU}{WU} = \dfrac{AV}{XV}$

E. all

9. Maya deposited 1000 dollars at 6% interest compounded annually. What is the number of dollars in the account after four years?

A. $1258.47   B. $1260.18   C. $1262.48

D. $1263.76   E. $1264.87

10. Let D be the determinant of the matrix A below:

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 \end{bmatrix}$$

A. $D = 0$ B. $D = 1$

C. $D = -1$ D. $D = 6$

E. none of these

11. If $0 < \theta < \pi$ and $2\cos^2\theta + \sin\theta - 2 = 0$, then the set of all possible values for θ is

A. $\{0\}$ B. $\left\{\frac{\pi}{6}\right\}$ C. $\left\{\frac{\pi}{4}, \frac{3\pi}{4}\right\}$

D. $\left\{\frac{\pi}{3}, \frac{2\pi}{3}\right\}$ E. $\left\{\frac{\pi}{6}, \frac{5\pi}{6}\right\}$

12. A class of 30 students occupies a classroom containing 5 rows of seats with 8 seats in each row. What is the probability that the 6th seat in the 5th row will remain empty, if the students sit at random throughout the seats in the classroom?

A. $\frac{1}{5}$ B. $\frac{1}{3}$ C. $\frac{1}{4}$ D. $\frac{1}{2}$ E. $\frac{1}{6}$

13. Consider the four statements below:

I. If a, b, and c are the sides of $\triangle ABC$, and $a^2 + b^2 = c^2$ then $\triangle ABC$ is a right triangle.

II. If $\triangle ABC$ is a right triangle with legs a and b and hypotenuse c, then $a^2 + b^2 = c^2$.

III. If $n > 2$ is a prime number than n is odd.

IV. If $n > 2$ is an odd number then n is a prime.

The only true statements in the above list are

A. I only

B. I and II only

C. I and III only

D. I, II, and III only

E. I, II, III and IV

14. A man has a pocket full of change, but cannot make change for a dollar. What is the greatest value of coins he could have?

A. $.99   B. $1.09   C. $1.19

D. $1.29   E. $1.39

UNC Charlotte HS Math Samples

15. Statistics have shown that in a certain college course, 65% of the students pass the first time they take it. Among those who have to repeat it, 70% pass on the second attempt, and among those who have to take it three times, 50% pass on the third attempt. What percentage of students have to take the course more than three times?

   A. 1%   B. 5.25%   C. 22.75%

   D. 35%   E. 50%

16. Solve the inequality $\dfrac{x+2}{2x-1} < 5$. The solution for x is

   A. $x < \frac{1}{2}$ or $\frac{7}{9} < x$ B. $x < 2$

   C. $\frac{1}{2} < x < 2$ D. $-\frac{1}{2} < x < 2$

   E. $\frac{1}{2} < x < \frac{7}{9}$

17. If $|3 - a| \le 7$ and $|b + 5| \le 9$, what is the smallest possible value of $a - b$?

   A. -18 B. -6 C. 0

   D. -4 E. -8

18. The student's in Professor Einstein's class decided to reward the fine teacher with a CD player at the end of the course. A total of $529 was collected from the students, with each student contributing the same amount, which was equal to the total number of students in the class. Only ordinary US bills were used and none of these were $2 bills. In addition, each student paid using the same five bills. How many ten-dollar bills were collected?

   A. 10   B. 12   C. 15   D. 23   E. 46

19. If the formula for Fahrenheit temperature in terms of Centigrade is $F = (\frac{9}{5})C + 32$, what is the Centigrade temperature when it is -18 degrees Fahrenheit?

   A. $\frac{70}{9}$ B. $\frac{126}{5}$

   C. -90 D. $-\frac{250}{9}$

   E. none of these

20. In the figure, $\overline{PA}$, $\overline{QB}$, and $\overline{RC}$ are each perpendicular to $\overline{AC}$. Which of the following is correct?
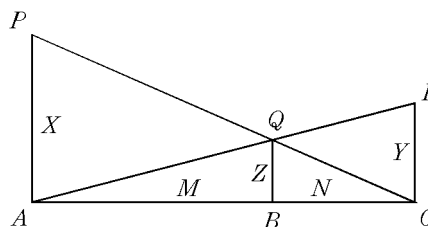
   A. $\dfrac{Z}{X} = \dfrac{N}{M}$ B. $\dfrac{Z}{X} = \dfrac{M+N}{N}$

   C. $\dfrac{Z}{Y} = \dfrac{M}{N}$ D. $\dfrac{Z}{Y} = \dfrac{M+N}{M}$

   E. $\dfrac{Z}{Y} = \dfrac{M}{M+N}$

21. Let n be the number of solutions of $\left|\sin(2x)\right| = \left|\cos(2x)\right|$ in the interval $0 \le x \le 2\pi$. Then

   A. $n \le 4$ B. $5 \le n \le 6$

   C. $6 \le n \le 7$ D. $8 \le n$

22. A family is traveling due west on a straight road that passes a famous landmark. At a given time the bearing to the landmark is $62°$ west of north. After the family has traveled 5 miles further, the landmark is $38°$ west of north. What is the closest the family can come to the landmark if they remain on the road (in miles, to the nearest $\frac{1}{100}$)?

A. 0.55   B. 3.54   C. 4.55

D. 5.46   E. 8.55

23. A sequence of three real numbers forms an arithmetic progression with a first term of 9. If 2 is added to the second term and 20 is added to the third term, the three resulting numbers form a geometric progression. What is the smallest possible value for the third term of the geometric progression?

A. 1   B. 4   C. 36   D. 49   E. 81

24. Three points are selected simultaneously and randomly from the 3 by 3 grid of lattice points shown. What is the probability that they are collinear? Express your answer as a common fraction.

$$\begin{matrix} \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet \end{matrix}$$
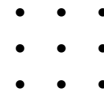
A. 1/42   B. 1/21   C. 2/21

D. 1/7   E. 1/6

25. How many pounds of coffee A beans with a price of \$2.08 a pound should be added to coffee B beans with a price of \$3.88 a pound to make 3 pounds of a coffee blend with a price of \$2.53 a pound?

A. 0.5   B. 0.75

C. 1.5   D. 2.25

E. none of these