

North Carolina State HS Math Samples

1) If a car's wheel is rotating 60 rev/min and radius of the wheel is 18 inches, how fast does the car travel in ft/sec?

- A. 3π B. $\frac{3\pi}{2}$ C. 36π
D. 18π E. 180π

2) A cubic foot of water weighs 62.5 pounds. Approximately how much would an 11-inch cube of water weigh?

- A. 48.1 lb B. 53.0 lb C. 53.7 lb
D. 58.6 lb E. 61.3 lb

3) Find the term containing t^5 in the expansion of $(2r - t)^7$.

- A. $84r^2t^5$ B. $-84r^2t^5$
C. $-21r^2t^5$ D. $-42r^2t^5$
E. none of these

4) Simplify: $(9x^{-6}y^4)^{-\frac{1}{2}}(2x^4y^{-5})^{-2}$

- A. $-\frac{3y^8}{4x^5}$ B. $\frac{y^8}{12x^6}$
C. $-\frac{1}{12x^{11}y^{12}}$ D. $-\frac{1x^5}{18y^{12}}$
E. none of these

5) $6x^2 - 6a^2$ factors completely as:

- A. $(3x + 2a)(2x - 3a)$
B. $6(x - a)(x - a)$
C. $(6x - a)(x + 6a)$
D. $6(x + a)(x - a)$
E. none of these

6) Let x and y be integers so that $3x + 7y = 1$. There are many solutions to this equation. Which of the following is the smallest positive value of $x + y$.

- A. 5 B. 7 C. 4 D. 3 E. 1

7) If y varies jointly as z and the square of x ; $y = 15$ when $z = 4$ and $x = 2$, find y when $z = 4$ and $x = 6$.

- A. 22.5 B. 45 C. 84.5
D. 115 E. 135

8) A function of the form $f(x) = \frac{a}{x+b}$ has the following properties $f(1) = 3$ and $f^{-1}(5) = -1$. What is the value of $f(0)$?

- A. $\frac{15}{4}$ B. $-\frac{3}{5}$
C. 4 D. $\frac{15}{7}$
E. none of these

9) Solve: $\left|5 - \frac{x}{3}\right| < 4$

- A. $x < -3$ or $x > 3$ B. $1 < x < 9$
 C. $x < -9$ or $x > 1$ D. $3 < x < 27$
 E. none of these

10) Simplify: $(x^{\sqrt{2}} - y^{\sqrt{2}})(x^{\sqrt{2}} + y^{\sqrt{2}})$

- A. $x - y$ B. $\sqrt{x} - \sqrt{y}$
 C. $2(x^{\sqrt{2}} - y^{\sqrt{2}})$ D. $x^{\sqrt{8}} - y^{\sqrt{8}}$
 E. none of these

11) Solve: $2x - 7\sqrt{x} - 15 = 0$

- A. \emptyset B. $\{25\}$ C. $\{5\}$
 D. $\left\{\frac{9}{4}, 5\right\}$ E. $\left\{-\frac{9}{4}, 5\right\}$

12) A pea weighs as much as three crumbs; five crumbs weigh as much as a pea and a kernel; a bean weighs as much as a pea and a crumb. If a kernel weighs 6 grams (g), how much does a bean weigh?

- A. 8 g B. 16 g C. 12 g
 D. 9 g E. 10 g

13) Solve for x : $\frac{b-a}{x} = \frac{b}{b+x} - \frac{a}{a+x}$

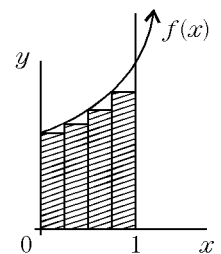
- A. $\frac{-ab}{a+b}$ B. $\frac{ab}{a+b}$
 C. $\frac{2ab}{a-b}$ D. $\frac{a+b}{a-b}$
 E. none of these

14) If $\frac{a-1}{a+1} = \frac{b-4}{b+3}$ then b equals:

- A. $\frac{7a+1}{2(1-a)}$ B. $\frac{a+1}{2}$ C. $\frac{2a}{a+1}$
 D. $\frac{7a+1}{2}$ E. $\frac{-7a-1}{2}$

15) In calculus, the area of a region under a curve is approximated by summing the areas of the rectangles shown. If the rectangles are equal in width and the function graphed is $f(x) = x^2 + 1$, determine the approximate area described.

- A. $\frac{11}{8}$ B. $\frac{19}{16}$
 C. $\frac{25}{16}$ D. $\frac{39}{32}$
 E. $\frac{47}{32}$



16) Find the sum of the values of m that make $f(x) = x^2 + (m+5)x + (5m+1)$ a perfect square trinomial.

- A. 3 B. 4 C. 7
 D. 8 E. 10

17) At 4:00 the angle between the minute hand and the hour hand on a standard analog clock is 120° . How much time will elapse until the two hands will again form an angle of 120° ?

- A. 40 min. B. 42 min.
 C. $43\frac{7}{11}$ min. D. $47\frac{7}{12}$ min.
 E. 60 min.

18) A square has its base on the x -axis, and one vertex on each branch of the curve $y = \frac{1}{x^2}$. What is its area?

- A. $2\sqrt[3]{2}$ B. $2\sqrt{2}$ C. 2
 D. 2.5 E. $\sqrt{7}$

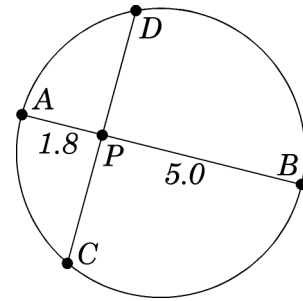
19) Given square $ABCD$ with sides 8 inches. A circle is drawn through vertices A and D , tangent to side BC . The radius of the circle is:

- A. 4 inches B. $4\sqrt{2}$ inches
 C. 5 inches D. $5\sqrt{2}$ inches
 E. 6 inches

20) A compass and straightedge construction begins with a square $ABCD$. The circle with center C and radius CD is constructed, intersecting line AC in a point E outside of the region bounded by the square. Some additional constructions occur, ending with a point F on the perpendicular bisector of segment AE such that angle CFE is one-third of angle AFE . What is the degree measure of angle AFE ?

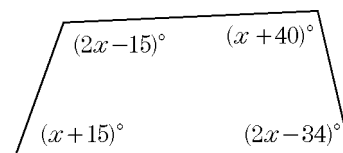
- A. 90 B. 105 C. 112.5
 D. 127.5 E. 135

21) Two perpendicular chords of a circle intersect at point P . If $AP = 1.8$ and $BP = 5.0$ determine CD .



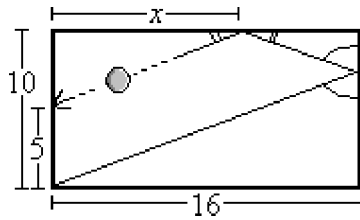
- A. 3 B. 5.2
 C. 6.0 D. 6.4
 E. None of these

22) Suppose a quadrilateral has angles as indicated in the diagram. What is the measure of the smallest angle?



- A. 59° B. 61° C. 74° D. 76°
 E. none of these

- 23) Given a $16\text{ ft} \times 10\text{ ft}$ room in which a ball is kicked against a wall, and eventually hits the midpoint of the opposite wall. Assuming that the angles from bounces are preserved and that the ball's path is as shown, find x .



- A. $7\frac{1}{2}\text{ ft}$ B. 8 ft
 C. 10 ft D. $10\frac{1}{2}\text{ ft}$
 E. none of these
- 24) The perimeter of an equilateral triangle is 24. The measure of an altitude of the triangle is:
- A. 4 B. $4\sqrt{3}$ C. $6\sqrt{2}$
 D. $6\sqrt{3}$ E. $8\sqrt{2}$
- 25) A metal flagpole on level ground broke at a certain point with the top part tipping over like a hinge and the tip hitting the ground at a point 20 ft from the base. It was rewelded but again broke, this time at a point 5 ft lower than before, with the tip hitting the ground at a point 30 ft from the base. The height of the flagpole is:
- A. 21 ft B. 29 ft C. 50 ft
 D. 53 ft E. 47.5 ft

- 26) The length, width, and height of a rectangular solid are in ratio of 3:4:12. If the diagonal of the solid is 39 inches long, what is the length of the longest dimension of the solid?

- A. 12 in. B. 48 in.
 C. 36 in. D. 72 in.
 E. none of these

- 27) To estimate the height of a tree, a logger measures $6\frac{1}{2}$ paces from the edge of the shadow thrown by the top of the tree to its base. He also notes that a 4 foot stick casts a $1\frac{1}{2}$ foot shadow. If the logger averages 4 feet per pace, then the height of the tree is approximately:

- A. 30 feet B. 40 feet C. 50 feet
 D. 60 feet E. 70 feet

- 28) George, lost in the dessert, starts at point A travels 4 km north, then turns and continues 3 km east, then turns southwest and continues $\sqrt{8}$ km and ends at point B. How far is point A from point B?

- A. 3 km B. $(5 - \sqrt{8})\text{ km}$
 C. $\sqrt{13}\text{ km}$ D. $\sqrt{17}\text{ km}$
 E. $\sqrt{5}\text{ km}$

- 29) A circle given by the equation $x^2 - 10x + y^2 + 2y + 10 = 0$. What are the coordinates of the center of this circle?

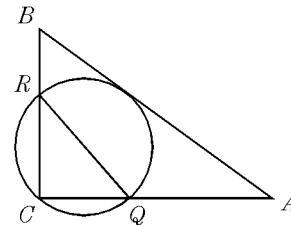
- A. (5, -2) B. (-2, 5) C. (5, -1)
 D. (-5, 1) E. (3, -1)

30) The point (a, b) is reflected over the y -axis to the point (c, d) , which is reflected over the x -axis to the point (e, f) . Compute $ab - ef$.

- A. 0 B. 2 C. $2ab$
 D. $2cd$ E. $-2cd$

31) In $\triangle ABC$, $AB = 13$, $AC = 12$, and $BC = 5$. Circle P is the circle with smallest radius which passes through C and is tangent to \overline{AB} . Let Q and R be the points of intersection, distinct from C , of circle P with sides \overline{AC} and \overline{BC} , respectively. The length of segment QR is:

- A. 2.6
 B. 5
 C. 5.2
 D. $\frac{60}{13}$
 E. $\frac{48}{13}$



32) A rectangular piece of paper is folded in half as shown in figure I below. If the two opposite corners of the folded paper are cut off as shown in figure II, which of the following is the design of the paper when unfolded? **No folding is permitted during testing.**

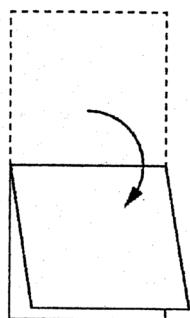


Figure I

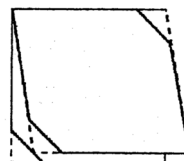
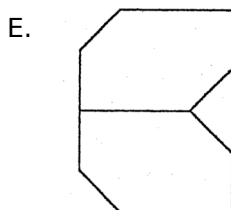
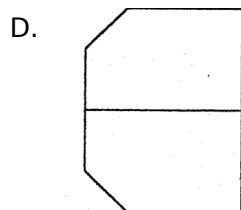
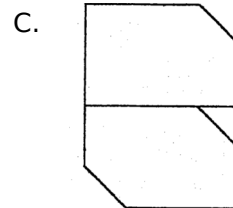
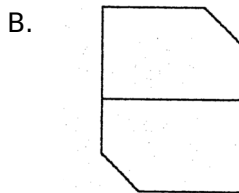
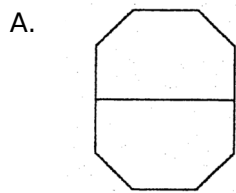


Figure II



33) Which one of the following statements is false?

- A. $\log_{10} \frac{10^7}{10^4} = 3 \log_{10} 10$
- B. $\log_{10} 4 + \log_{10} 9 = \log_{10} 13$
- C. If $\log_b N = L$, then $b^L = N$
- D. $\log_2 16 = 4$
- E. none of these

34) Let z_j for $1 \leq j \leq 5$ denote the five distinct fifth roots of 1 in the complex plane. Let $w_j = \frac{z_j}{1+z_j^2} + \frac{z_j^2}{1+z_j^4} + \frac{z_j^3}{1+z_j} + \frac{z_j^4}{1+z_j^3}$ for $1 \leq j \leq 5$. How many distinct values belong to the collection $\{w_j : 1 \leq j \leq 5\}$?

35) Give the determinant of the following matrix: $\begin{bmatrix} 3 & 5 \\ 2 & 4 \end{bmatrix}$

- A. -2
- B. -1
- C. 0
- D. 1
- E. 2

36) Find the x -coordinate of the vertex of the parabola $x = y^2 + y + 1$.

- A. -0.5
- B. 0.75
- C. 1.25
- D. 1.75
- E. none of these

37) A , B , and C are the vertices of an isosceles right triangle with right angle at B . The area of this triangle is 1. Point E on side AC of this triangle is such that segments AE and AB have the same length. Point D on side BC of this triangle is such that the segment DE is perpendicular to side AC . The area of the triangle whose vertices are A , D , and C is:

- A. $\frac{1}{2}$
- B. $\sqrt{2}$
- C. $\frac{1}{\sqrt{2}}$
- D. $2 - \sqrt{2}$
- E. $\sqrt{2} - 1$

38) A man walks x miles due west, turns 150° to his left and walks three miles in the new direction. If he finishes at a point $\sqrt{3}$ miles from his starting point, then x is:

- A. $\sqrt{3}$
- B. $2\sqrt{3}$
- C. $\frac{3}{2}$
- D. 3
- E. not uniquely determined by the given information

39) The function $f(x)$ is defined by $f(x) = \cos^4(x) + K \cos^2(2x) + \sin^4(x)$ where K is a constant. If the function $f(x)$ is a constant function, what is the value of K ?

- A. -1
- B. $-\frac{1}{2}$
- C. 0
- D. $\frac{1}{2}$
- E. 1

40) Suppose the weight of a culture of bacteria doubles daily. If the weight of the culture at the end of k days is M grams, its weight at the end of $k - 3$ days is:

- A. $\frac{M}{3}$
- B. $\frac{M}{8}$
- C. $8M$
- D. $\frac{M}{9}$
- E. none of these

41) Mr. Smith is dividing students into groups for a school project. If he arranges them into groups of 2, 3, 4, 5, or 6, there is always one student left over. What is the smallest number of students he could have?

- A. 21 B. 60 C. 61
D. 720 E. 721

42) Which one of the following statements is true if all the rest are false?

- A. Statements 'a', 'b', and 'c' are all false.
B. Statement 'e' is true.
C. Statements 'b' or 'c' or 'd' is true.
D. If 'a' is false then 'c' is false.
E. Both 'a' and 'c' are false.

43) Troy's test average (based on 5 exams) is 83.2, and he wants to raise his grades to an 85. What is the minimum score he needs on the sixth test to achieve his goal?

- A. 87 B. 90
C. 94 D. 98
E. none of these

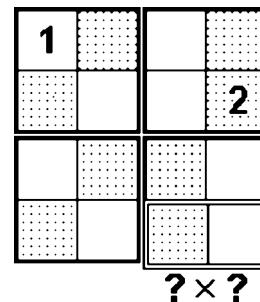
44) The average age of 5 people in a room is 30 years. An 18-year-old person leaves the room. What is the average age of the four remaining people?

- A. 25 B. 26 C. 29
D. 33 E. 36

45) Given the following information: Of the 135,000 voters 56% were women. 52% of the women and 47.5% of the men voted for the Democrat. How many votes did the Democrat get?

- A. 67,527 B. 71,145
C. 75,600 D. 75,222
E. None of these

46) Consider this modified Sudoku puzzle. The numbers 1 through 4 appear in every column, every row, and every 2×2 exactly once. In addition a shaded square will contain an even number, while an un-shaded square will contain an odd one. Find the product of the values in the two right-most squares of the bottom row.



- A. 2 B. 4 C. 6
D. 8 E. 12

- 47) Which of the following properties of real numbers is illustrated by:

$$ab + (ac + cd) = a(b + c) + ad$$

- A. Commutative property of addition
- B. Associative property of multiplication
- C. Associative property of addition and the associative property of multiplication
- D. Associative property of addition and the distributive property
- E. none of these

- 48) The numbers 1, $3 = 1 + 2$, $6 = 1 + 2 + 3$, ... are examples of triangular numbers. More generally a positive integer is a triangular number if it can be written as the sum of the first n consecutive positive integers for some value of n . How many of the positive integers 1, 2, 3, ..., 1998 are triangular numbers?

- 49) A farmer's corral measured 20' wide by 24' long. He needed more space, so he increased the area of the corral 75%. If the length was increased by 25%, what was the width increased?

- A. 50% B. 28% C. 25% D. 40%
- E. none of these

- 50) When a ball drops from any height it bounces and travels upward a distance that is $\frac{1}{3}$ the previous height. If the ball is dropped, bounces back up and down, and continues to bounce up and down so that it has traveled a total distance of 106 feet when it strikes the ground for the fourth time, what was the original height from which it was dropped?

- A. 2 feet B. 54 feet
- C. 71.55 feet D. 79.5 feet
- E. none of these