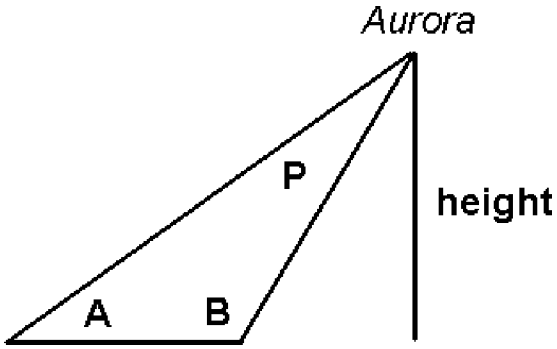
How High is an Aurora?

Image courtesy Tim Tomljanovich, http://www.nsaclub.org/photos/aurora/

For thousands of years, people living at northern latitudes had no idea how high up the Aurora Borealis was located. Before the advent of photography in the 1880's, auroral observers tried to determine the height of aurora by the method of triangulation. One of the earliest of these measurements was made by the French scientist Jean-Jacques d'Ortous de Mairan between 1731 and 1751. From two stations 20 km apart, observers measured the angles A and B between the ground and a specific spot on an aurora. From the geometry of the triangle, they estimated that aurora's height was between 650–1,000 km above the ground. More precise measurements yielded estimates from 70 to 200 kilometers.



1. Suppose that two observers were located 30 kilometers apart. Observer A measured an angle of 53 degrees and Observer B measured an angle of 114 degrees. By making a scaled drawing of this triangle, what was the height of the auroral feature they were studying?

2. Use a protractor to measure the vertex angle, *P*. What happens to the measurement of angle *P* if you decrease the 'baseline' distance between the observers to 5 kilometers?

3. What would the measurements of the two angles be if the aurora were located over a spot half-way between the two observers?

4. The sun produces 3.9×10^{33} ergs per second of radiant energy. How much energy does it produce in one year (3.1×10^{7} seconds)?

5. One gram of matter converted into energy yields 3.0×10^{20} ergs of energy. How many tons of matter in the sun is annihilated every second to produce its luminosity of 3.9×10^{33} ergs per second? (One metric ton = 10^{6} grams)

6. The mass of the sun is 1.98×10^{33} grams. If a single proton has a mass of 1.6×10^{-24} grams, how many protons are in the sun?

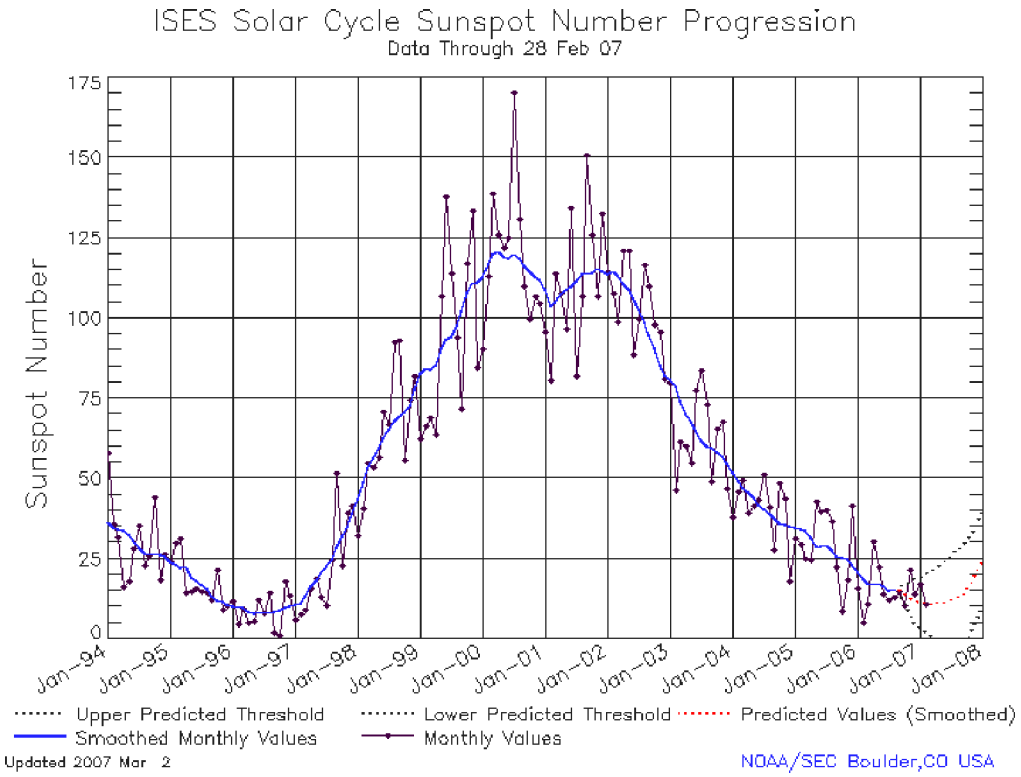
7. The approximate volume of the visible universe (A sphere with a radius of about 14 billion light years) is 1.1×10^{31} cubic light-years. If a light-year equals 9.2×10^{17} centimeters, how many cubic centimeters does the visible universe occupy?

8. The NASA data archive at the Goddard Space Flight Center contains 25 terabytes of data from over 1000 science missions and investigations. (1 terabyte = 10^{15} bytes). How many CD-roms does this equal if the capacity of a CD-rom is about 6×10^{8} bytes? How long would it take, in years, to transfer this data by a dial-up modem operating at 56,000 bits/second? (Note: one byte = 8 bits).

9. Pluto is located at a distance of 5.9×10^{14} centimeters from Earth. At the speed of light (2.99×10^{10} cm/sec) how long does it take a light signal (or radio message) to travel to Pluto and return?

10. The planet HD209458b, now known as Osiris, was discovered by astronomers in 1999 and is at a distance of 150 light-years (1 light-year = 9.2×10^{12} kilometers). If an interstellar probe were sent to investigate this world up close, traveling at a maximum speed of 700 km/sec (about 10 times faster than our fastest spacecraft: Helios-1), how long would it take to reach Osiris?

The Sunspot Cycle - endings and beginnings



The above plot shows the current sunspot cycle (Number 23) based on the average monthly sunspot counts since January, 1994.

11. During which part of the sunspot cycle is there

    1. the greatest month-to-month variation in the number of sunspots counted?
    2. The least variation in the number counted?

12. About when (month and year) did Sunspot Cycle 23 begin?

13. a) What was the average minimum sunspot count during the years of the previous sunspot minimum?

    b) What do you think the average sunspot count will be during the current sunspot minimum?

14. When (year, month) do you think the next sunspot minimum will occur?

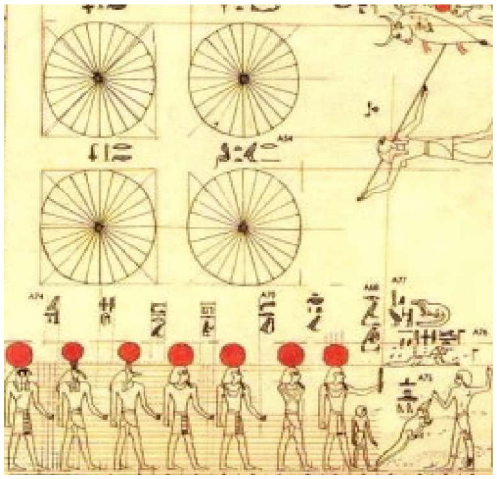
15. a) What was the average minimum sunspot count during the years of the previous sunspot minimum?

    b) What do you think the average sunspot count will be during the current sunspot minimum?

16. During which part of the sunspot cycle is there

    1. the greatest month-to-month variation in the number of sunspots counted?
    2. The least variation in the number counted?

17. **Scientific Notation II**



Astronomers rely on scientific notation in order to work with 'big' things in the universe. The rules for using this notation are pretty straightforward, and are commonly taught in most 7th-grade math classes as part of the National Education Standards for Mathematics.

The following problems involve the addition and subtraction of numbers expressed in Scientific Notation. For example:

$$1.34 \times 10^8 + 4.5 \times 10^6 = 134.0 \times 10^6 + 4.5 \times 10^6$$
$$= (134.0 + 4.5) \times 10^6$$
$$= 138.5 \times 10^6$$
$$= 1.385 \times 10^8$$

1. $1.34 \times 10^{14} + 1.3 \times 10^{12} =$
2. $9.7821 \times 10^{-17} + 3.14 \times 10^{-18} =$
3. $4.29754 \times 10^3 + 1.34 \times 10^2 =$
4. $7.523 \times 10^{25} - 6.32 \times 10^{22} + 1.34 \times 10^{24} =$
5. $6.5 \times 10^{-67} - 3.1 \times 10^{-65} =$
6. $3.872 \times 10^{11} - 2.874 \times 10^{13} =$
7. $8.713 \times 10^{-15} + 8.713 \times 10^{-17} =$
8. $1.245 \times 10^2 - 5.1 \times 10^{-1} =$
9. $3.64567 \times 10^{137} - 4.305 \times 10^{135} + 1.856 \times 10^{136} =$
10. $1.765 \times 10^4 - 3.492 \times 10^2 + 3.159 \times 10^{-1} =$

A 'Galactic City' in the Far Reaches of our Universe

Astronomers have recently discovered a massive cluster of young galaxies that formed when the universe was only about 1 billion years old. The light from these galaxies has taken over 12 billion years to reach Earth. The growing galactic metropolis, called COSMOS-AzTEC3 is the most distant known massive "proto-cluster" of galaxies known today. The circled red smudges in the above image are the individual galaxies that are a part of the cluster. In the time since the light started on its journey, these galaxies have probably fallen together under the influence of gravity to form a large galaxy about the size of our Milky Way. At the distance of the galaxy cluster, the width of this photograph, which was taken by the Japanese Subaru Telescope on Mauna Kea, is about 25 million light years across. (Image Credit: Subaru/ NASA / JPL-Caltech)

18. Assuming all of the galaxies lie in the same plane, and at the same distance, to two significant figures:

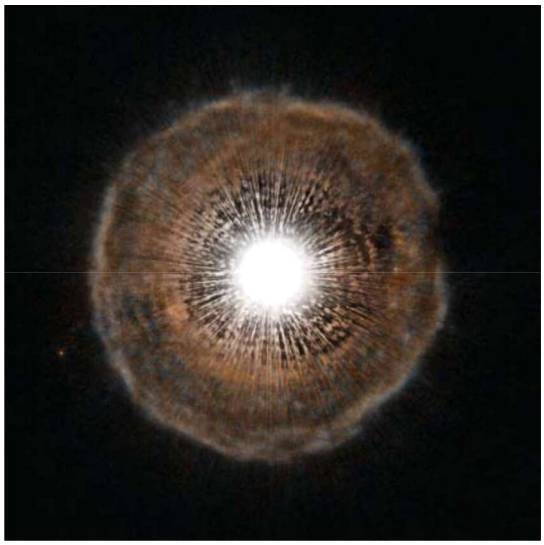
    a) What is the greatest distance between the galaxies in this photograph?

    b) What is the smallest distance between galaxies in this photograph?

19. The Milky Way galaxy and the Andromeda galaxy are separated by about 2,200,000 light years in diameter. How many millimeters apart would they appear if they were viewed at the same distance as this cluster?

20. If the average speed of a galaxy in this cluster is about 1000 light years in 1 million years, how many years will it take for all the galaxies to fall to the center of the cluster?

21. **The Expanding Gas Shell of U Camelopardalis**



This dramatic image taken by the Hubble Space Telescope reveals details in the shell of gas ejected by the star U Camelopardalis thousands of years ago.

Located 1,400 light years from our sun, the shell is expanding at a speed of about 25 km/sec, and its outer edge is about 500 billion km from the central star, whose image has been greatly over exposed making it seem huge in this image!

Although it looks impressive, the amount of mass in this shell is actually quite small. It is only about $\frac{1}{10}$ the mass of our own planet Earth!

The radius of the shell is 500 billion kilometers, and the estimated speed is about 25 km/sec. How many years did it take for the shell to get this big if 1 year = 31 million seconds?

22. The mass of Earth is 6.0×10^{24} kg. The mass of a hydrogen atom is 1.6×10^{-27} kg. If the entire mass of the shell were evenly spread out in a sphere with the shell's radius, how many hydrogen atoms would you expect to find in a cubic meter of this shell to the nearest 10,000 atoms?

23. **Estimating the Mass of a Cloud!**

You look up at the sky one day and see puffy little cumulus clouds hovering over the beach, a meadow, or over your town. Did you ever wonder just how much a cloud might weigh as it drifts by over your head?

Different clouds carry different amounts of water droplets and so they have different densities. Brilliant white cumulus clouds, for example, have densities of 0.3 grams/meter^3.

From the known cloud densities, we can estimate their masses once we know their volumes because Mass = Density × Volume.

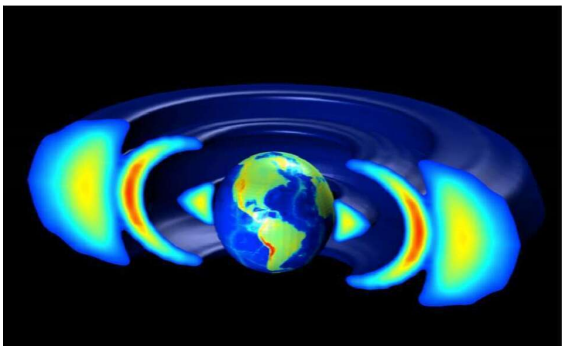
a) From the definition of density, what are the other two equations you can create that define mass and volume?

b) A puffy cumulus cloud looks almost like a sphere. If its diameter is 3.0 kilometers, what is its volume in cubic meters? (use $\pi = 3.14$)

c) What is the total mass of the cumulus cloud in kilograms and metric tons?

d) You spot two clouds in the sky. The cumulus cloud is $\frac{1}{5}$ the diameter of the cumulonimbus cloud, and the cumulonimbus cloud has 8 times the density of the cumulus cloud. What is the ratio of the mass of the cumulus cloud to the cumulonimbus cloud if both clouds are spherical in shape?

24. **Exploring the Third Belt with the Van Allen Probes**



(Image credit: Yuri Shprits, Adam Kellerman, Dmitri Subbotin/UCLA)

The Van Allen Probes spacecraft travel in an elliptical orbit through the Van Allen belts. Soon after launch, they detected a third radiation belt shown by the middle crescent in the figure to the left.

In this problem, we predict when the spacecraft will encounter this third belt along their orbit, so that scientists can schedule observations of this new region.

The equation for the orbit of the spacecraft is given in Standard Form by

$$1 = \frac{x^2}{6.25} + \frac{y^2}{9.0}$$

The equation for the location of the third Van Allen belt projected into the plane of the elliptical orbit and concentric with the orbit focus centered on Earth $(0, -2)$ is given by

$$5.8 = x^2 + (y + 2)^2$$

On the same coordinate plane, graph these two functions and estimate the coordinates of the intersection points to the nearest tenth.

25. Using only algebra, find the coordinates of all intersection points between the orbit and the new belt region to the nearest tenth.